Using immediate feedback to enhance learning in higher education

Participants

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Main principles:

- Autonomy   (Freedom of choise)
- Competence/Mastering
- Belonging (Affiliation)

It was important for us to:

- listen to the students (student influence)
- try different solutions to try to come up with the best possible one. We made all the time changes based on feedbacks from the students.
- in the prossess conducted surveys to find out what solution the students liked best and why.

An important background for our study was the article:

Formative assessment and self-regulated learning: a model and seven principles of good feedback practice
David J. Nicol & Debra Macfarlane-Dick

We tried to adapt this principles to a best possible fit for our classes, with focus on formative learning.

Immediate feedback was of crucial importance for us.

We wanted
- to use conceptual questions to increase understanding
- The students to feel on competence/mastering

Introduction
This study is part of a larger PHD-study, studying the use of digital aids in mathematics for Bachelor Students in Trondheim.

Several of our colleges had been working with **two** Student Respons Systems developed at our institute, one giving **single quizzes (called SRS)** and another making **several quizzes** possible in the same answering process (**called PeLe**).

In both systems we use **multiple choice problems** and the student can **answer via mobile, pad or computer.**

Our study started in august 2016, there have been two courses each year, both for 1. year students, Mathematics 1 (autumn semester) and Mathematics 2 (spring semester) and two classes with together around 180-190 students.

The PHD-study is about the whole courses, but in this study we only focus on the exercise program.

When we started we wanted to implement for Bachelor Students the exercise program used at the preliminary courses given at our institute.

Experiences, goals and contact with the students gave a process that led to several changes before we finally ended up with an exercise program that we ourselves feel good about.

Responses from the students by both students conversations and by response from a group of 3-4 students sitting in a so called reference group, that are having three meetings with the teacher each semester, **all indicates that the students also feel that this exercise program is good.**

The last year, 2018/2019, we have used this final program on our students. Thus the study presented here is only based on the last year.

Background

Together with working out mathematical problems in groups and having a weekly possibility to get help from a student assistant, as in the ordinary exercise program, **we wanted to give them a deeper understanding of the mathematical concepts and context**, and **our goal was to achieve this by having exercises where they first of all are given immediate responses of their works and further are given the possibility to study related or similar problems in a class situation lead by the teacher.**

We then needed **a system that can give the teacher an immediate response from the students answers**, and we created follow-up questions to the different tasks in the exercises, that we could give to the students in the class.

This question should be conceptual, and the students were encouraged to see the connections and try to see what really happens without doing any further calculus.

Main aspects are **student activity, cooperation and also learning by speaking/discussing mathematics.**

Giving feedback to and accepting feedback from other students are important, so we wanted to challenge the students to this, by participating in the discussions.

Explaining something for others are a very good way to learn and getting feedback from fellow students can give another and useful perspective.

Our main focus have been on learning and the goal was to achieve enhanced learning by using conceptual questions and student activity.

Summarized, we wanted our exercise program to

- increase student activity
- learn the students to cooperate
- give students learning from discussions
- be interactive with immediate response
- emphasize learning

Setup for the exercises

We decided that 6 of 10 exercises should be of the new type.
Since we are using the answering system called PeLe, we call them PeLe-exercises.

The other 4 are what we call ordinary exercises.

The students need some time to work with the exercises by themselves or in groups. The exercises are published on the Learning Management System (LMS) in the beginning of the semester, so the students can start working with them whenever they want to.

They are also given the possibility to get help from a student assistant 2 hours each week. They are working in groups of 3-5 persons in these sections.

The above is common for all the exercises.

The ordinary exercises are then answered groupwise with scanned documents sent to the LMS (Learning Management System). The student assistant then registrate them and give some simple corrections. If the answers are good enough they get the exercise approved.

The PeLe-exercises are a little more complex. There we also have 2 x 45 minutes in a classroom with the teacher present. We call this a PeLe-class, and the whole further process can be summarized as this:

- Alternatives for each task are presented two days before the PeLe-class. **We want the students to work by solving the exercises first before we present the exercises**, and not only to look after an alternative that fits.

- **The code for giving the digital answer to the PeLe-exercise is given two days before the PeLe-class.** The students answers this by choosing the alternatives that they find correct. The deadline for this is the start of the PeLe-class. An important thing here is the possibility to show what tasks they have found difficult by flagging the task.

- The PeLe class starts by **looking at the results of the exercise.** The students are asked to sit down together with their working-group. This is for the discussion parts of the class. As shown in figure 1.1 the results are easily seen and based on this the program for the PeLe-class is planned. During the PeLe class

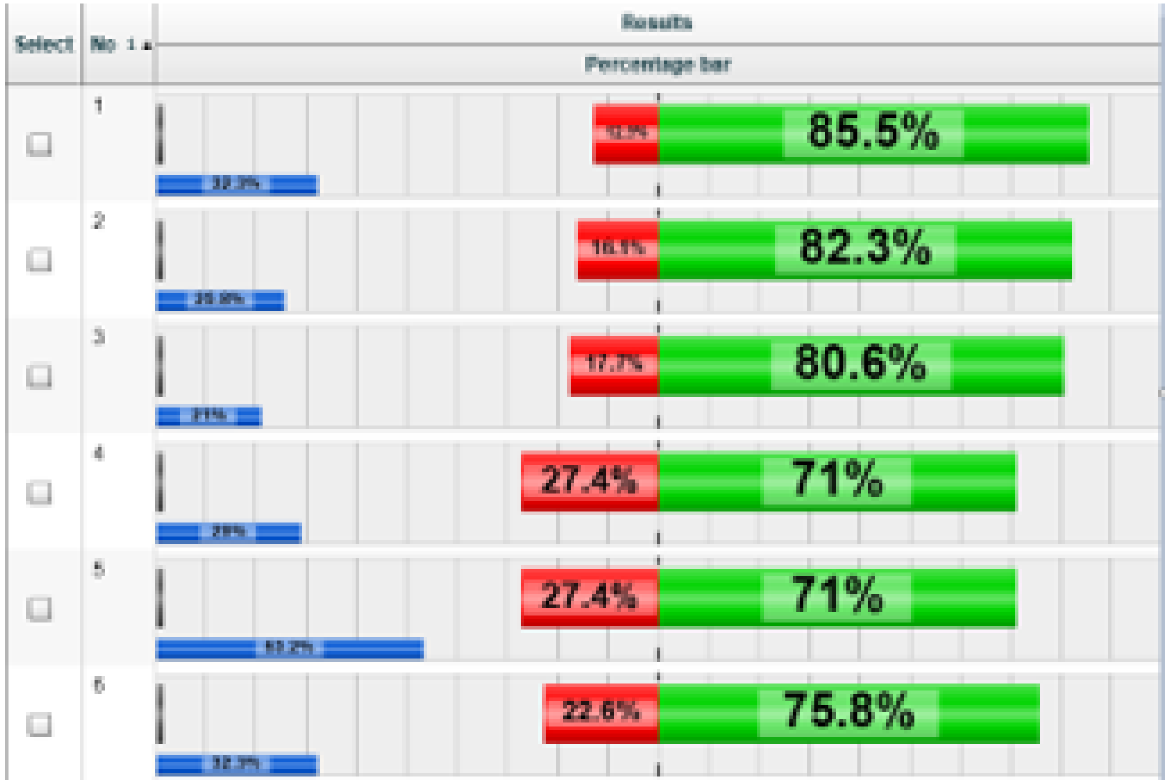
    - **The teacher give a solution to all the exercises.** The results shows which exercises that needs more or less attention. Little time is used on the best answered tasks, more on the tasks with not so good results.

    * **Follow-up questions (one or more) are given at chosen tasks.** The students are given some time to discuss each question, before they give an electronical answer. The result is immediately seen, and gives background for how much time the teacher spend on explanation. If the result is bad, the teacher can give a tip and do a revote.

    * At the end of the PeLe-class the students deliver their written answers to the teacher. They can complement their answers during all the PeLe-class before they deliver.

To get the exercise approved the student need to deliver both the digital and written answer and also give digital answers to the follow-up questions (this is because we want the students to participate in the PeLe-class). In the written answer they also need at least to have a good try on each of the tasks. They get their answers back in the next week.

Fig 1.1 Example showing how the results on the exercise are shown. Here there are 6 questions. The green and red bar shows respectively how many percent of the students that have given the correct and wrong answer. Since there can be students logged in that are not completing their answers the number do not sum up to 100%. The blue bars shows how many students that have flagged each task.



RESULTS
At the end of the second semester, a web-based survey covering 12 different aspects comparing the new vs. the old exercise program was conducted for all the students who followed the two mathematics courses.

The survey was completely anonymous and voluntary. About 45% of students, i.e. approximately 90 out of 200 students, completed the survey.

The main objective of the survey was to investigate how the students experienced their own learning process with this new exercise program compared to the traditional exercise program.

One of the main objectives of this new program was to improve the students' learning effect by:

i) **increasing the contact between the lecturer and the student**

ii) **making the exercise program more discussion-based about conceptual issues as a basis for increased insight into computational-based issues**

iii) **giving the students direct and more targeted feedback.**

The various questions in the survey therefore addressed the students' own view of whether the new exercise program improved their own learning process, and whether it gave them increased motivation to continue working with the subject matter on their own.

The new exercise program

Survey	Completely agree (%)	Agree to some extent (%)	Disagree to some extent (%)	Completely disagree (%)
The PeLe exercise program has worked well for me	64,8	31,8	3,4	0,0
The PeLe exercise program is a good exercise program	63,6	31,8	4,6	0,0
I learn more from the PeLe exercise program than the traditional exercises program	49,4	33,3	16,1	1,2

Feedback and presented answers

Survey	Completely agree (%)	Agree to some extent (%)	Disagree to some extent (%)	Completely disagree (%)
Immediate feedback from the lecturer is important for my learning	72,4	25,3	2,3	0,0
It is helpful with a quick review of the solution of each exercise during the PeLe-lessons	73,6	20,7	3,4	2,3

Understanding

Survey	Completely agree (%)	Agree to some extent (%)	Disagree to some extent (%)	Completely disagree (%)
The associated quiz exercises improve my understanding of the mathematical concepts	55,2	29,9	11,5	3,4
The associated quiz exercises improve my understanding of mathematical relationships	54,0	31,0	11,5	3,5

Learning and clearing up misunderstandings

Survey	Completely agree (%)	Agree to some extent (%)	Disagree to some extent (%)	Completely disagree (%)
I learn from discussing the associated quiz exercises with other students	55,2	33,3	9,2	2,3
I learn by thinking through each associated quiz exercise on my own	57,5	33,3	9,2	0,0
The associated quiz exercises help me to clear up misunderstandings	57,5	36,8	3,4	2,3

Attending classes

Survey	Completely agree (%)	Agree to some extent (%)	Disagree to some extent (%)	Completely disagree (%)
I would have attended most PeLe lessons even though I didn't have to	48,3	35,6	14,9	1,2

Examples

A typical task may thus be as follows:

Let $f(x, y)$ be as in task 5. Determine the level curves corresponding to the function

$$z = f(x, y) = \frac{1}{\sqrt{x^2 + y^2 - 9}} = k \tag{1}$$

when $k = 1$, $k = \frac{1}{4}$ and $k = \frac{1}{\sqrt{7}}$.

As in the traditional approach the students may in the middle of the week attend a time scheduled exercise-hour led by a student assistant. The student assistant guides the students according to the traditional approach. Shortly after this exercise-hour the lecturer publishes several possible solutions to each task. For the task given above these options may look as the list given below:

A. Straight lines with slopes equal to 1, $\frac{1}{4}$ and $\frac{1}{\sqrt{7}}$ respectively.
B. Parabolas that are symmetrical about the y-axis and which intersects the y-axis at 1, $\frac{1}{4}$ and $\frac{1}{\sqrt{7}}$ respectively.
C. Parabolas that are symmetrical about the x-axis and which intersects the x-axis at 1, $\frac{1}{4}$ and $\frac{1}{\sqrt{7}}$ respectively.
D. Circles with centers at the origin and radii equal to 1, $\frac{1}{4}$ and $\frac{1}{\sqrt{7}}$ respectively.
E. Circles with centers at the origin and radii equal to $\sqrt{10}$, 5 and 4 respectively.
F. Hyperbolas with the x and y axis as asymptotes and passing through the points $(1,1)$, $\left(1, \frac{1}{4}\right)$, and $\left(1, \frac{1}{\sqrt{7}}\right)$.
G. Ellipses with center in the origin and with half axis equal to $a = 1 \;\wedge\; b = 1$, $a = 1 \;\wedge\; b = \frac{1}{4}$ and $a = 1 \;\wedge\; b = \frac{1}{\sqrt{7}}$.
H. My answer does not match any of the above alternatives

A. Straight lines with slopes equal to $1, \frac{1}{4}$ and $\frac{1}{\sqrt{7}}$ respectively.

B. Parabolas that are symmetrical about the y-axis and which intersects the y-axis at $1, \frac{1}{4}$ and $\frac{1}{\sqrt{7}}$ respectively.

C. Parabolas that are symmetrical about the x-axis and which intersects the x-axis at $1, \frac{1}{4}$ and $\frac{1}{\sqrt{7}}$ respectively.

D. Circles with centers at the origin and radii equal to $1, \frac{1}{4}$ and $\frac{1}{\sqrt{7}}$ respectively.

E. Circles with centers at the origin and radii equal to $\sqrt{10}$, 5 and 4 respectively.

F. Hyperbolas with the x and y axis as asymptotes and passing through the points $(1,1)$, $\left(1,\frac{1}{4}\right)$, and $\left(1,\frac{1}{\sqrt{7}}\right)$.

G. Ellipses with center in the origin and with half axis equal to $a = 1 \;\wedge\; b = 1$, $a = 1 \;\wedge\; b = \frac{1}{4}$ and $a = 1 \;\wedge\; b = \frac{1}{\sqrt{7}}$.

H. My answer does not match any of the above alternatives

Related to the task above such a quiz may be given as shown below:

We found in exercise 7 with the function

$$f(x,y) = \frac{1}{\sqrt{x^2+y^2-9}} \tag{2}$$

that the three specified level curves were given by alternative E. What happens to the corresponding level curves if the function is changed to

$$g(x,y) = \frac{5}{\sqrt{x^2+y^2-9}} \quad ? \tag{3}$$

A. They become circles that are closer together

B. They become circles that are equally close

C. They become circles that are farther away from each other

D. They won't be circles anymore

E. I don't know

Join at **menti.com** use code **32 26 47 5**

SRS til oppgave 7

Vi fant i oppgave 7 at med

$$f(x,y) = \frac{1}{\sqrt{x^2 + y^2 - 9}}$$

er de tre angitte nivåkurvene gitt ved alternativ E.

Hva skjer med nivåkurvene om funksjonen endres til

$$f(x,y) = \frac{5}{\sqrt{x^2 + y^2 - 9}} \ ?$$

A. De blir sirkler som ligger tettere.

B. De blir sirkler som ligger like tett

C. De blir sirkler som ligger mindre tett

D. De blir ikke sirkler lenger

E. Vet ikke

De partielle derivert blir nå 5 ganger større, dvs. vi har en brattere graf.
Dette vil gi kortere avstand mellom nivåkurvene.
Tenk kart: Brattere terreng ⇒ Kotene ligger tettere. A.

SRS nr. 20: Diagonalisering

Matrisen $$A = \begin{bmatrix} 7 & 2 \\ 3 & 8 \end{bmatrix}$$

Har egenverdiene $\lambda_1 = 10$ og $\lambda_2 = 5$, med tilhørende egenvektorer

$$\vec{x} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} s \ \ , s \neq 0 \quad \text{og} \quad \vec{x} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} t \ \ , t \neq 0$$

A kan da diagonaliseres til D ved hjelp av matrisen P, det vil si: $P^{-1}AP = D$, der

A. $D = \begin{bmatrix} 10 & 0 \\ 0 & 5 \end{bmatrix}$ og $P = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

B. $D = \begin{bmatrix} 5 & 0 \\ 0 & 10 \end{bmatrix}$ og $P = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$

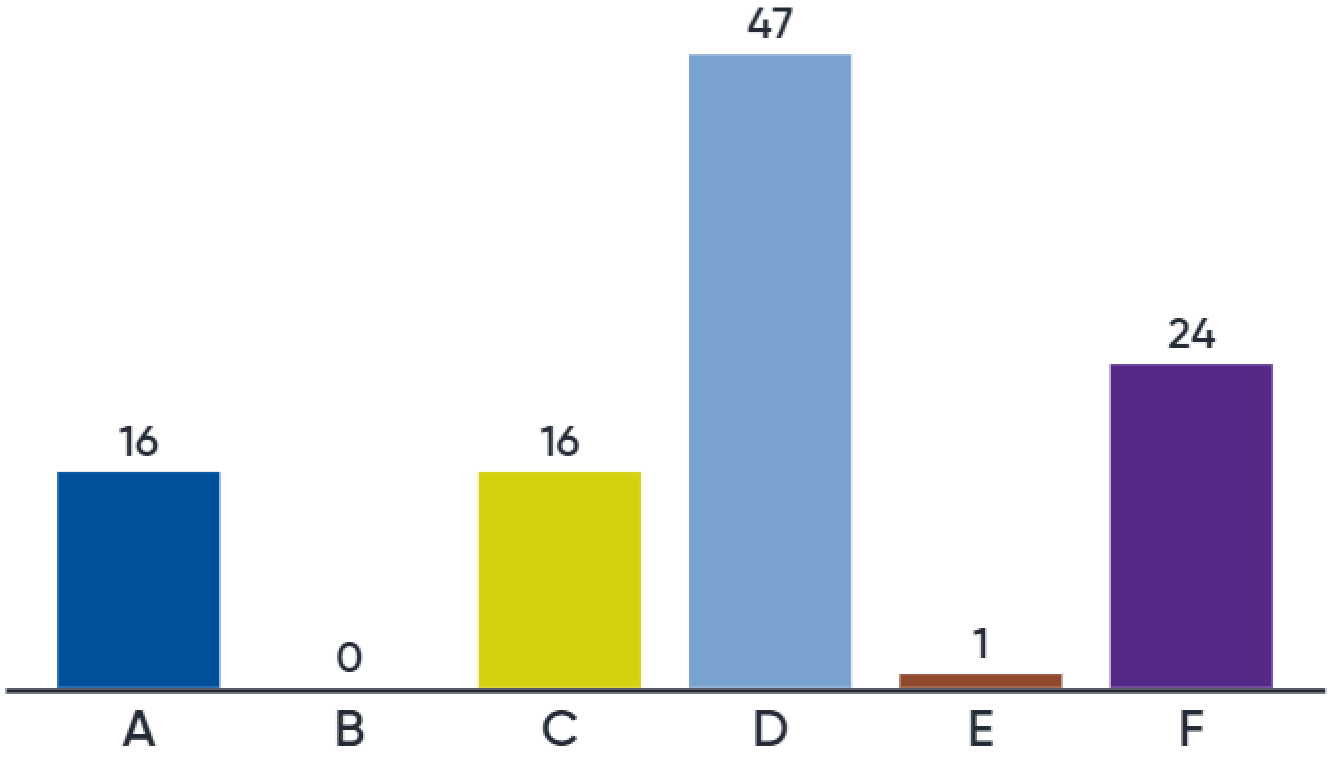
C. $D = \begin{bmatrix} 10 & 0 \\ 0 & 5 \end{bmatrix}$ og $P = \begin{bmatrix} 2 & -1 \\ -3 & 1 \end{bmatrix}$

D. $D = \begin{bmatrix} 5 & 0 \\ 0 & 10 \end{bmatrix}$ og $P = \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}$

E. Ingen av delene.

F. Vet ikke

Hvilket alternativ er korrekt?



Oppgave 1

Gitt

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$$

Bestem en matrise P og en diagonal matrise D slik at $P^{-1}AP = D$.

SRS 1 til oppgave 1

Vi fant følgende i oppgave 1: for at $P^{-1}AP = D$ skal gjelde må vi benytte

$$\boldsymbol{D} = \begin{bmatrix} -\mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{5} \end{bmatrix}$$

$$\boldsymbol{P} = \begin{bmatrix} \mathbf{3} & -\mathbf{1} \\ -\mathbf{3} & -\mathbf{2} \end{bmatrix}$$

Vil det også fungere å benytte

$$D = \begin{bmatrix} -1 & 0 \\ 0 & 5 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & -3 \\ 2 & 3 \end{bmatrix} \ ?$$

A) JA

B) NEI

C) Vet ikke

Oppgave 4

Summen av tre positive tall er 100. Finn tallene når produktet av dem er størst mulig.

A. 22, 32, 46

B. 22, 36, 42

C. 23, 29 ,48

D. 27, 35, 43

E. 30, 30, 40

F. 30, 35, 35

G. 34, 38, 29

H. 35, 36, 31

I. 39, 42, 19

J. Mitt svar passer ikke med noen av alternativene over

SRS 2 til oppgave 4

Vi fant i oppgave 4 at x + y + z = 100 ga størst mulig produkt $P = x \cdot y \cdot z$ for x = y = z.

Om vi krever at x + 2y + 3z = 100, vil vi da få størst mulig produkt $P = x \cdot 2y \cdot 3z$ for x = 2y = 3z?

A) JA

B) NEI

C) Vet ikke

Oppgave 4

Anta at hvert år skifter 30% av eiere av biler med tohjulstrekk til bil med firehjulstrekk, mens 10% av eierne av bil med firehjulstrekk skifter til bil med tohjulstrekk.
Det totale antallet biler er konstant, og hver bileier har kun en bil.
Dersom 25% av bileierne har firehjulstrekk nå, hvor mange prosent av bileierne har firehjulstrekk om 10 år? Hva vil andelen som har firehjulstrekk gå mot når $k \to \infty$?

A. 50 % og 60 % B. 58,7 % og 70 % C. 68,2 % og 72 % D. 74,7 % og 75 %

E. 77,2 % og 80 % F. Mitt svar passet ingen av de øvrige svaralternativene

SRS 1 til oppgave 4

Hva ville andelen som har firehjulstrekk gått mot på lang sikt $(k \rightarrow \infty)$ dersom 40 % av bileierne hadde firehjulstrekk nå?

A) 40 % B) 45 % C) 50 % D) 55 % E) 60 % F) 65 % G) 70 %

H) 75 % I) 80 % J) Ingen av delene K) Vet ikke

Oppgave 2
La

$$f(x,y) = 3 + xy - x - 5y.$$

Finn det kritiske punktet til f, og bestem dets type.

A. $(0,1)$, Minimalpunkt
B. $(0,1)$, Sadelpunkt
C. $(1,0)$, Minimalpunkt
D. $(1,0)$, Sadelpunkt
E. $(1,2)$, Maksimalpunkt
F. $(1,2)$, Minimalpunkt
G. $(5,1)$, Maksimalpunkt
H. $(5,1)$, Sadelpunkt
I. $(1,5)$, Maksimalpunkt
J. Mitt svar passer ikke med noen av alternativene over

SRS 2 til oppgave 2

Vi fant i oppgave 2 at $f(x, y) = 3 + xy - x - 5y$ har kun ett kritisk punkt og at dette er punktet (5, 1) som er et sadelpunkt.

Erstatter vi leddet 3 med 7 slik at vi får funksjonen
$g(x, y) = 7 + xy - x - 5y$, får vi

A) (5, 1) er fortsatt eneste kritiske punkt og fortsatt et sadelpunkt.

B) (5,1) er fortsatt eneste kritiske punkt, men er ikke et sadelpunkt.

C) kun ett kritisk punkt, men nå ikke (5,1) og dette er et sadelpunkt.

D) kun ett kritisk punkt, men nå ikke (5,1) og dette er ikke et sadelpunkt.

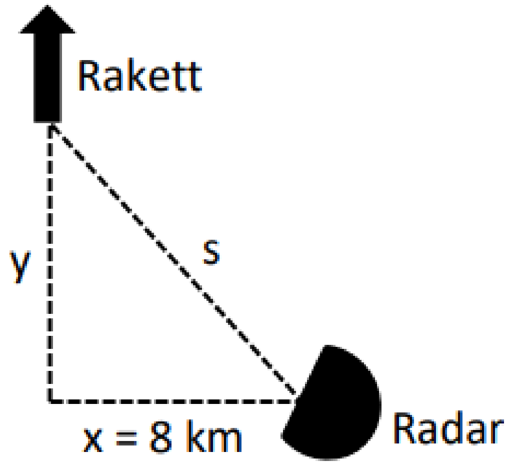
E) mer enn ett kritisk punkt.

F) ingen kritiske punkter.

G) Vet ikke.

Oppgave 3

En romsonde sendes vertikalt opp fra jorden på tur til planeten Venus. Den observeres fra bakken ved hjelp av en radar. Horisontal avstand fra oppskytningsrampen til radaren er 8 km, se figuren under:



På et bestemt tidspunkt så er $s = 17\,\text{km}$ og

$$\frac{ds}{dt} = 100 \text{ km/min}.$$

Bestem hastigheten til raketten i y-retning, v_y, på akkurat dette tidspunktet.

- I. 53,3 km/min
- J. 88,2 km/min
- K. 100 km/min
- L. 112,8 km/min
- M. 113,3 km/min
- N. 1692 km/min
- O. 1700 km/min
- P. Mitt svar passet ingen av de øvrige svaralternativene

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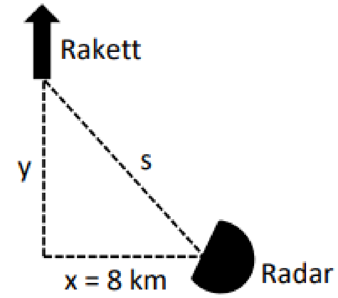

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SRS 1 til oppgave 3

Hva når $s = 20\,\text{km}$? Anta fortsatt $\frac{ds}{dt} = 100\ \text{km/min}.$ Vil $\frac{dy}{dt}$ da bli

A. Større

B. Lik

C. Mindre

D. Vet ikke

SRS 1 til oppgave 3

Hva når $s = 20\,\text{km}$? Anta fortsatt $\frac{ds}{dt} = 100$ km/min. Vil $\frac{dy}{dt}$ da bli

A. Større

B. Lik

C. Mindre

D. Vet ikke

Formelen for $y'(t)$ endres ikke der

$$y'(t) = \frac{s(t) \cdot s'(t)}{y(t)} = \frac{s(t)}{y(t)} \cdot s'(t)$$

Når s øker vil også y øke.

La θ = vinkelen ved radaren. Da er $\sin\theta = \frac{y}{s}$.

Vi ser at θ øker når y øker $\Rightarrow \sin\theta$ øker

$\Rightarrow \frac{s}{y} = \frac{1}{\sin\theta}$ avtar

Siden $s'(t)$ er den samme

har vi da at $y'(t)$ avtar

Alternativ

C

Oppgave 3

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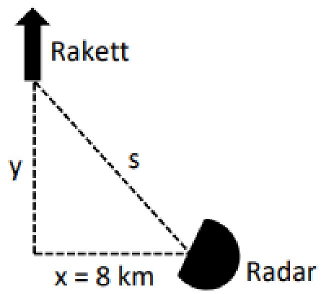

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SRS 2 til oppgave 3

Hva om vi observerer fra et punkt lenger unna? La x = 9 km og s = 17 km og sett fortsatt $\frac{ds}{dt} = 100\ \text{km/min}.$ Vil $\frac{dy}{dt}$ da bli

A. Større

B. Lik

C. Mindre

D. Vet ikke

SRS 2 til oppgave 3

Hva om vi observerer fra et punkt lenger unna? La x = 9 km og s = 17 km og sett fortsatt $\frac{ds}{dt} = 100$ km/min. Vil $\frac{dy}{dt}$ da bli

A. Større

B. Lik

C. Mindre

D. Vet ikke

Formelen for $y'(t)$ endres ikke der

$$y'(t) = \frac{s(t)\cdot s'(t)}{y(t)}$$

s er den samme, så x øker $\Rightarrow$ y avtar

Telleren holdes konstant og nevneren avtar

$\Downarrow$

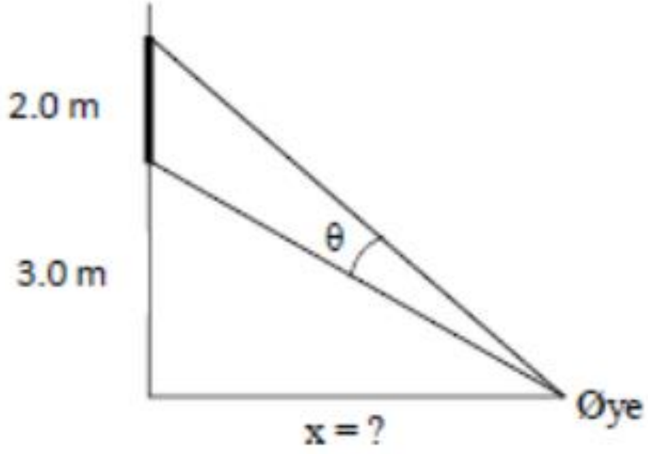
$y'(t)$ øker

Alternativ A

Oppgave 5

Et bilde, som er 2.0 m høyt, henger på en vertikal vegg slik at nedre billedkant er 3.0 m over øyenivået til en person.

Hvor langt fra veggen skal personen stå for at synsvinkelen θ skal bli størst mulig?



Tips: Finn først θ som funksjon av avstanden x ved f.eks. å betrakte θ som differensen mellom to vinkler og benytte arctan-funksjonen.

A. $x = 3$

B. $x = \sqrt{10}$

C. $x = \sqrt{15}$

D. $x = \sqrt{18}$

E. $x = 4,5$

F. $x = \sqrt{21}$

G. $x = 6$

H. Mitt svar passet ingen av de øvrige svaralternativene

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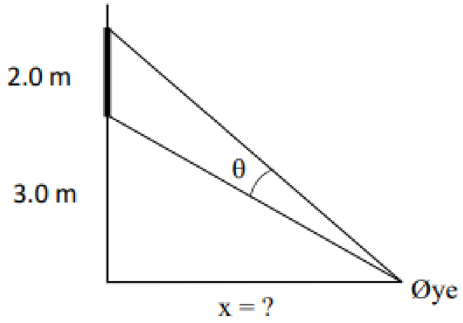

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H. Mitt svar passet ingen av de øvrige svaralternativene

SRS 1 til oppgave 5

En person står en gitt avstand fra maleriet. Maleriet flyttes slik at høyden over bakken nå blir 2 m.

Synsvinkelen vil da bli

A. Større

B. Lik

C. Mindre

D. Vet ikke

SRS 1 til oppgave 5

En person står en gitt avstand fra maleriet. Maleriet flyttes slik at høyden over bakken nå blir 2 m.

Synsvinkelen vil da bli

A. Større

B. Lik

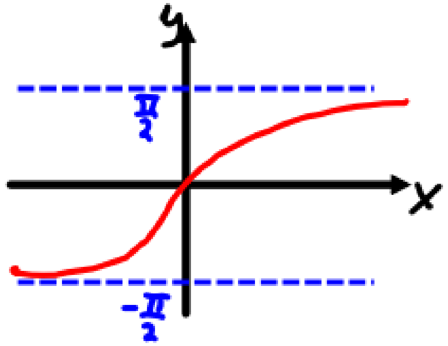
C. Mindre

D. Vet ikke

Vi får nå $\theta = \arctan\left(\frac{4}{x}\right) - \arctan\left(\frac{2}{x}\right)$

Siden $(\arctan x)' = \frac{1}{1+x^2}$ ser vi at stigningstallet til grafen avtar når x øker, $x \in [0, \rightarrow\rangle$ (se også figur under.)

Da vil $\arctan a - \arctan b$, der $a-b$ er konstant, bli mindre jo større a (og dermed også b) er.



Vi hadde $\theta = \arctan\left(\frac{5}{x}\right) - \arctan\left(\frac{3}{x}\right)$, og endret til $\theta = \arctan\left(\frac{4}{x}\right) - \arctan\left(\frac{2}{x}\right)$

Vi endret altså fra $a = \frac{5}{x}, b = \frac{3}{x}$ til $a = \frac{4}{x}, b = \frac{2}{x}$.

($a - b = \frac{2}{x}$ = konst. for hver verdi av x.)

a (og dermed også b) minker

$\Rightarrow \theta = \arctan a - \arctan b$ øker.

For en hver gitt avstand x, vil dermed synsvinkelen θ bli større.

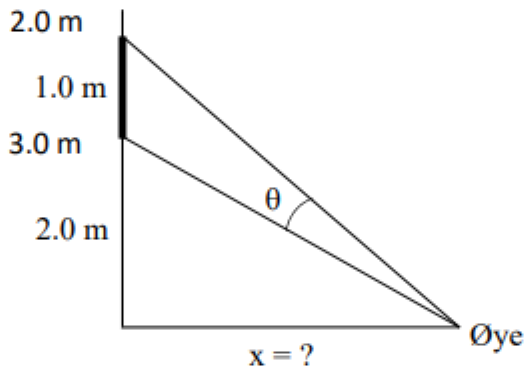
Alternativ Ⓒ

Oppgave 5

Et bilde, som er 2.0 m høyt, henger på en vertikal vegg slik at nedre billedkant er 3.0 m over øyenivået til en person.

Hvor langt fra veggen skal personen stå for at synsvinkelen θ skal bli størst mulig?

Tips: Finn først θ som funksjon av avstanden x ved f.eks. å betrakte θ som differensen mellom to vinkler og benytte arctan-funksjonen.



SRS 2 til oppgave 5

Hva skjer med denne avstanden x dersom

1) høyden av bildet øker, men det skal ha samme avstand til bakken?
2) bildet henges høyere, dvs. avstanden fra bildet til bakken øker?

La Ø stå for at x øker, S stå for at x blir den samme og M stå for at x minker.

Riktig svar for henholdsvis 1) og 2) er:

A. Ø - Ø
B. Ø - S
C. Ø - M
D. S - Ø
E. S - S
F. S - M
G. M - Ø
H. M - S
I. M - M
J. Vet ikke

SRS 2 til oppgave 5

Hva skjer med denne avstanden x dersom

1) høyden av bildet øker, men det skal ha samme avstand til bakken?
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La Ø stå for at x øker, S stå for at x blir den samme og M stå for at x minker.

Riktig svar for henholdsvis 1) og 2) er:

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- D. S - Ø
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- G. M - Ø
- H. M - S
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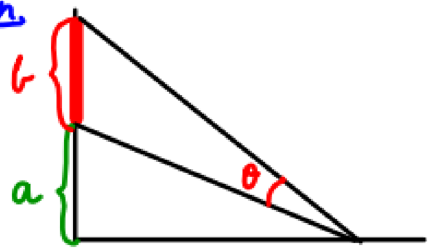
- J. Vet ikke

Løsn.



$$\theta = \arctan\left(\frac{a+b}{x}\right) - \arctan\left(\frac{a}{x}\right)$$

$$\frac{d\theta}{dt} = \frac{-\frac{a+b}{x^2}}{1+\left(\frac{a+b}{x}\right)^2} + \frac{\frac{a}{x^2}}{1+\left(\frac{a}{x}\right)^2}$$

$$\frac{d\theta}{dt} = 0 \Rightarrow \frac{\frac{a}{x^2}}{1+\frac{a^2}{x^2}} = \frac{\frac{a+b}{x^2}}{1+\frac{(a+b)^2}{x^2}}$$

Vi utvider begge brøkene med x^2 og får: $\frac{a}{x^2+a^2} = \frac{a+b}{x^2+(a+b)^2}$

$$\Rightarrow ax^2 + a(a+b)^2 = (a+b)x^2 + (a+b)\cdot a^2$$

$$\Rightarrow a\cdot(a+b)\cdot[(a+b)-a] = [(a+b)-a]\,x^2 \Rightarrow \cancel{b}x^2 = a\cancel{b}\cdot(a+b)$$

$$\Rightarrow \underline{x = \sqrt{a\cdot(a+b)} = \sqrt{a^2+ab}}$$

Vi ser derfor at: <u>a øker</u> ⇒ <u>x øker</u>; <u>b øker</u> ⇒ <u>x øker</u>

<u>Alt. A er dermed rett.</u>